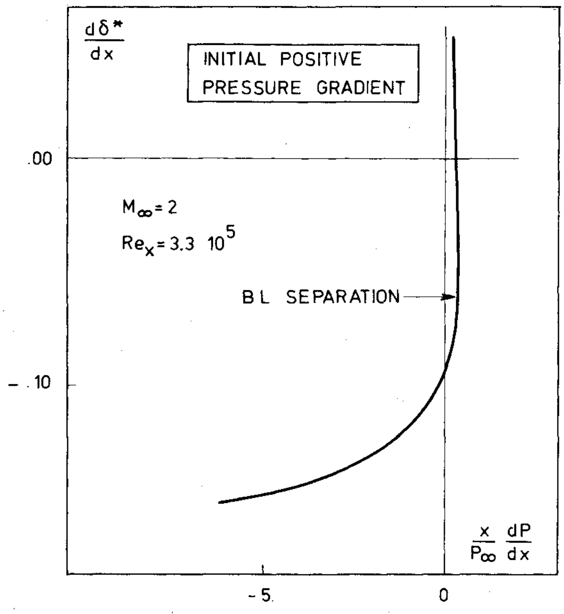


**Fig. 3 Relation between $d\delta^*/dx$ and $dp/dx$ for initial positive pressure gradient.**

2) Conversely for flows with gradient $p<0$ the derivative d grad $p/d\theta$ becomes very large. Hence a small variation in the imposed condition $\theta(x)$ or $\delta(x)$ may induce large differences in the result and the computation may fail if $\theta$ is such that $dp/dx \rightarrow -\infty$. Thus a direct method of computation seems more convenient if the boundary layer is expected to accelerate.

The ultimate method of computation appears to be a "mixed" method where the applied condition is neither $\delta^*(x)$ nor $p(x)$ but a linear combination of the two. A convenient linear relation appears to be: $\delta^*(x)+Ap(x)=B$ where $A$ is a function of the slope $d\delta^*/dp(x)$.

## Conclusion

A relation between the quantities $p(x)$ and $\delta^*(x)$ which may be prescribed in a boundary-layer calculation by direct or inverse method of solution is computed. The typical aspect of the curve $d\delta^*/dx=f(dp/dx)$ strongly supports the fact that inverse methods should be used to compute retarded or separated flows, but that direct method of solution is highly preferable for negative pressure gradients. A "mixed," inverse/direct method is now applied to calculate boundary-layer flows in various conditions.

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# Turbulent Boundary-Layer Calculations in Adverse Pressure Gradient Flows

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## Introduction

ALGEBRAIC eddy viscosity models are known to be inadequate for computing flows with adverse pressure gradients, as found on airfoils near the trailing edge. It may be some time before higher-order turbulence models are more accurate and reliable than this simpler model for engineering purposes, and the computational resources required for the use of higher-order models often prohibit their use for all but relatively simple geometries. Thus, this investigation was undertaken to determine if the range of usefulness of the two-layer algebraic eddy viscosity model could be extended to supersonic flows nearing separation. Numerical solutions of the turbulent boundary-layer equations are compared with the experimental data of Laderman,[1] Sturek,[2] and Waltrup and Schetz,[3] with the objective of increasing the accuracy of the two-layer eddy viscosity model for flows encountering constant, adverse pressure gradients by adjusting the three constants therein to obtain agreement with experiment.

## Numerical Procedure

The compressible, turbulent boundary-layer equations were cast in nondimensional form by Shang et al.,[4] using the algebraic eddy viscosity model with its extension to the apparent heat flux term and the Levy-Lees transformation. Three multilayer eddy viscosity models were evaluated by these authors for zero pressure gradient, adiabatic flows over a range of Mach numbers, and Reynolds numbers. They found that the Cebeci et al.[5] eddy viscosity model was more accurate over a wide range of flow conditions. The details of this method are contained in Ref. 4.

In this model, the equivalent eddy viscosity, $\epsilon$, is written as $\nu+\Gamma\epsilon_i$ in the inner region, consisting of the viscous sublayer and wall regions, and as $\nu+\Gamma\epsilon_0$ in the outer wake region, where $\nu$ is the molecular kinematic viscosity. The function $\Gamma$ represents the transition model of Dahawan and Narasimha as developed by Shang et al.,[4] and was zero for laminar flow and unity for fully turbulent flow. The present calculations were started with a laminar boundary-layer solution at the wind tunnel throat or boundary-layer origin, and transition was initiated when the length Reynolds number exceeded 0.5 million.

The eddy viscosity in the inner region was

$$\epsilon_i = k_1^2 y^2 D^2 \left| \frac{\partial u}{\partial y} \right|$$

where $D$ is the Van Driest damping factor. In the outer region the model was

$$\epsilon_0 = k_2 u_e \delta_i^* \gamma(y/\delta)$$

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where $\gamma$ is the intermittency correction factor of Cebeci et al.[6] and $\delta_i^*$ is the incompressible or kinematic boundary-layer thickness. Here, $u$ is the velocity in the $x$ or streamwise direction, $y$ is the coordinate normal to the streamwise direction, and $\delta$ is the value of $y$ when $u=0.995U_e$, the edge or freestream value.

The above constants, $k_l$, $A^+$, and $k_2$, known as the von Kármán constant, the sublayer thickness, and the Clauser constant, respectively, conventionally have values of 0.4, 26, and 0.0168, respectively, and determination of the variation of these constants with adverse pressure gradient was the specific objective of this investigation. The values of $k_l$ and $A^+$ were immediately raised from their upstream values to their new values upon encountering the pressure gradient at $(X/L)_I$. The value of $k_2$ varied linearly with streamwise distance, i.e., $k_2=k_e+k_s\ (X/L)-(X/L)_I$, where $k_e$ was the upstream or equilibrium value and $k_s$ the slope.

## Experimental and Numerical Data

Three sets of experimental data taken by Laderman,[1] Sturek,[2] and Waltrup and Schetz[3] were analyzed using the proven boundary-layer algorithm of Shang et al.[4] Similarities among these sets of data include supersonic Mach number, high Reynolds number, and the use of isentropic compression ramps to generate constant adverse pressure gradients. These mean flows were two dimensional, with adiabatic wall, and represent the known data for this type of flow.

The wall static pressure data, in each case, were fitted by a linear regression analysis to determine the slope and intercept.

### A. Laderman's Experiment

This experiment provided the most recent detailed measurements and velocity, and the initial motivation for this paper. The data showed that $k_l=0.65$—in agreement with the earlier data of Sturek, but considerably higher than the conventional value of 0.4. The complete experimental results are contained in Laderman's report,[1] and the numerical and experimental results relevant to this paper are shown in Figs. 1-3. The compression ramps begin at the nondimensional length $X/L=0.8$ and terminate at 1.056 (ramp 1) and 0.952 (ramp 3). The length of ramp 3 was limited by blockage in the $7.87\times8.64$ cm test section. The approaching boundary layer was 0.7 cm thick.

The skin friction, normalized by freestream dynamic pressure, is shown for both ramps in Fig. 1. The eddy viscosity model with the universal constants considerably underpredicts the skin friction (dotted line) as the pressure gradient is encountered. The new values for constants $k_l$ and $k_2$ were obtained by fitting Laderman's[1] experimental data, and the damping constant $A^+$ was varied until the agreement with experiment (solid curve) was obtained. This fitting procedure resulted in the values for the eddy viscosity constants for these ramps shown in Table 1. The skin friction increases with adverse pressure gradient, as shown by the experimental data and both viscosity models; however, the predictions using the universal constants are considerably smaller than the experimental results for both ramps. The surprising result of increasing skin friction with adverse pressure gradients has also been found in other experiments and calculations.

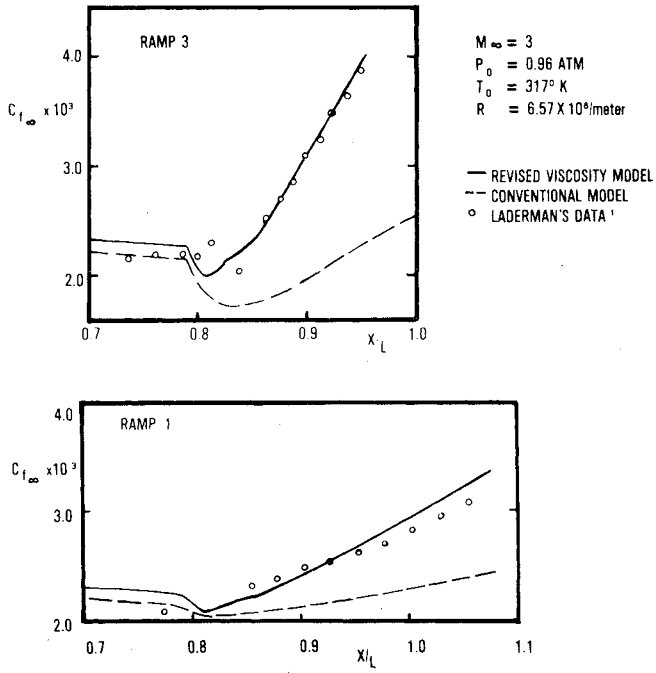


**Fig. 1 Skin friction.**

The validity of the revised eddy viscosity constants was tested by comparing the experimental velocity profiles with the calculated velocity profiles, as shown in Fig. 2 for ramp 1 and Fig. 3 for ramp 3. Ten velocity profiles were calculated and plotted along each ramp and the eight shown are representative. Computations using the conventional model for velocity profiles (dotted curves) progressively underpredict the data as the flow proceeds along the ramp. The predicted velocity profiles of the eddy viscosity model with the revised constants (solid curves) agree with the experimental data at all stations along both ramps.

### B. Sturek's Experiment

Sturek[2] also took data in a continuous flow research tunnel. His boundary layer developed along a 2.92 m flat plate that formed the floor of the tunnel. The asymmetric nozzle of the tunnel was formed by the upper wall, as opposed to the symmetric nozzle of Laderman's experiment, and caused the effective boundary-layer origin to occur at the plate leading edge.

**Table 1 Revised viscosity model coefficients**

| | Laderman[1] | | | Sturek[2] | | | | Waltrup and Schetz[3] | |
|---|---|---|---|---|---|---|---|---|---|
| | Upstream | Ramp 1 | Ramp 3 | Upstream | | | | Upstream | Case 2 |
| $M_\infty$ | 3 | 3 | 3 | 3.54 | 3.54 | 3.54 | 3.54 | 2.36 | 2.36 |
| $Re\times10^6$ | 3.31 | 3.31 | 3.31 | — | 43.13 | 53.92 | 64.70 | 32.58 | 32.58 |
| $\dfrac{d(P/P_\infty)}{d(X/L)}$ | 0 | 3.02 | 12.58 | 0 | 29.22 | 29.56 | 30.17 | 0 | 6.38 |
| mmHg/cm | 0 | 1.2 | 5 | 0 | 2.48 | 3.13 | 3.84 | 0 | 14 |
| $k_l$ | 0.40 | 0.65 | 0.65 | 0.40 | 0.65 | 0.65 | 0.65 | 0.40 | 0.65 |
| $A^+$ | 26 | 52 | 43 | 26 | 54 | 55 | 56 | 26 | 65 |
| $k_e$ | 0.02 | 0.02 | 0.02 | 0.01 | 0.01 | 0.01 | 0.01 | 0.0168 | 0.0168 |
| $k_s$ | 0 | 0.01 | 0.02 | 0 | 0.45 | 0.45 | 0.45 | 0 | 0.05 |
| $\delta/h$ | 0.0810 | — | — | 0.1667 | — | — | — | 0.0444 | — |

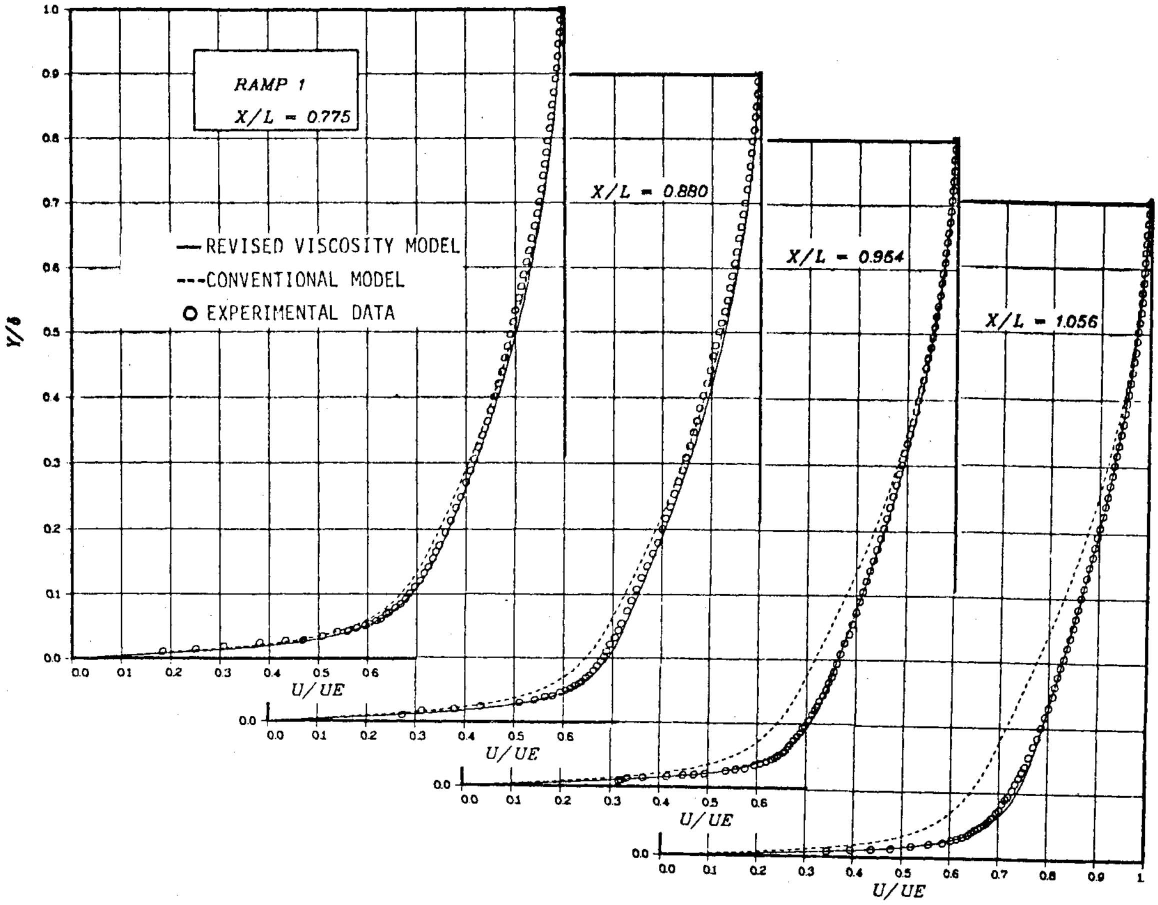


Fig. 2 Velocity profiles, ramp 1.

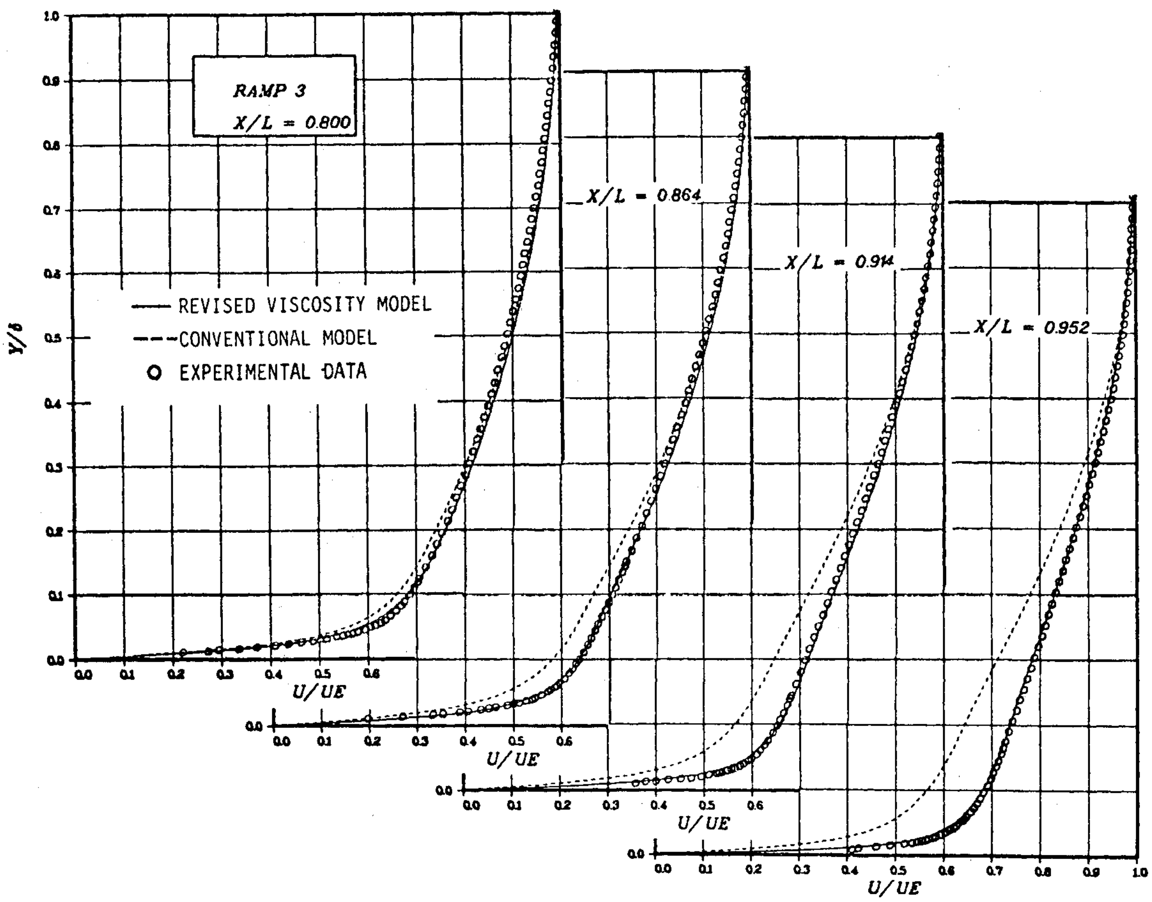


Fig. 3 Velocity profiles, ramp 3.

The data comparisons were similar to the previous cases. The normalized skin friction increased by a factor of 1.7 over its zero pressure gradient value and the conventional eddy viscosity model again considerably underpredicted this increase. Velocity profiles were compared and again good agreement with the data obtained. The agreement deteriorated slightly as the flow proceeded downstream; however, the revised model, in all cases, was in considerably better agreement with the data than the conventional model. A summary of the revised constants used in the pressure gradient region for the three values of Reynolds number is given in Table 1.

The value of $k_I$ increased to 0.65 immediately upon encountering the pressure gradient, as in Laderman's case, independent of streamwise distance and Reynolds number within the limited range of values of these test parameters. The damping constant $A^+$ appears to be independent of streamwise distance and not independent of Reynolds number, while the Clauser constant $k_2$ increases with streamwise distance.

### C. Waltrup and Schetz's Experiment

Unlike the previous cases, Waltrup and Schetz[3] induced three pressure gradients on the flat wind-tunnel floor using sting-mounted compression surfaces along the tunnel centerline. These pressure gradients were not constant. The skin friction also increased for this experiment, characteristic of entering an adverse pressure gradient region, and poorer predictions of the conventional viscosity model were evident. The boundary-layer thickness was predicted as well by either viscosity model when the experimental thickness was taken as the point where $u = 0.995\ U_e$, in agreement with the definition used for the computations. The calculated velocity profiles were in good agreement in the outer wake portion; however, the conventional viscosity model performed as well as the revised model. Neither model predicted the experimental results in the inner or wall region. Various scaling factors and many values of $k_I$, $A^+$, and $k_2$ were tried, but agreement with these data points near the wall could not be obtained.

## Conclusions

The constants in the algebraic eddy viscosity models are not constant for flow with adverse gradients. These calculations show that the inner layer reacts immediately through the constants $k_I$ and $A^+$ to the imposed pressure gradient, while the outer layer alone requires a lag equation (varying with streamwise distance) to permit the constant $k_2$ to attain its maximum value.

At present, a correlation of the values of $k_I$, $A^+$, and $k_2$ with physical properties of the flow has not been achieved. The values obtained to date are shown in Table 1. It is evident that additional experimental data will be necessary. Additional values of these constants, for various flows, may enable verification and extension of known correlations and rate equations or new correlations with a wider range of applicability to be determined. Alternatively, the scatter in these values may preclude correlations, in which case, the limitations of this model will have been defined and an advanced model will have to be tested.

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# Limiting Particle Streamline of Gas Particle Mixtures in Axially Symmetric Nozzles

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## Introduction

MUCH work has been done to estimate phase nonequilibrium effects on the performance of rocket engines. In general, however, an estimate of such an effect is very difficult, even in the quasi-one-dimensional approximation, and is often almost impossible, at least theoretically, for multidimensional flows. Much of the previous work, therefore, is primarily concerned with the quasi-one-dimensional flows.[1] There have been only a few studies of two-dimensional or axially symmetric nozzle flows.[2]

One of the complications in the gas particle flows arises because of the inertia of the solid particles. The particle streamlines may deviate from those of the gas, and their trajectories are determined by following the individual particles through the flowfield of the gas. Since the particles exert no pressure, particle trajectories near a nozzle wall are unaware of and uninfluenced by the presence of the wall boundary, suggesting that the particle streamlines near the wall are not required to be parallel to the wall. The particles may, therefore, detach from the wall or impinge on the wall. In the flows with particle detachment from the wall, there exists a particle-free region between the so-called limiting particle streamline and the wall boundary. The appearance of such a particle-free region in a flow introduces a mathematical difficulty. Also, the shape and location of the limiting streamline and the location of the point of the particle impingement on the wall are very important to the nozzle performance.

In this Note, particle behavior in axially symmetric supersonic nozzles is considered under the condition that the velocity and temperature lags are small. Particular attention is paid to the particle streamlines, especially limiting streamlines. The first-order problem is solved for particle streamlines.

## Coordinate Systems and Basic Equations

The dimensionless quantities are introduced by

$$x = \bar{x}/\bar{L},\ y = \bar{y}/\bar{r},\ \rho = \bar{\rho}/\bar{\rho}_*,\ p = \bar{p}/\bar{p}_*,\ V = \bar{V}/\bar{a}_*,\ T = \bar{T}/\bar{T}_*$$

$$\rho_p = \bar{\rho}_p/\bar{\rho}_{p*},\ V_p = \bar{V}_p/\bar{a}_*,\ T_p = \bar{T}_p/\bar{T}_* \qquad (1)$$

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